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The effect of ambient temperature on the propagation of nonadiabatic combustion waves

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In this paper, we undertake an analytical and numerical investigation of the linear stability and properties of travelling nonadiabatic combustion wave for the case of non-zero ambient temperature. Here we consider premixed fuel with one-step exothermic reaction described by Arrhenius law. The speed of the front is estimated analytically by employing the matched asymptotic expansion approach and numerically using the shooting and relaxation methods. It is shown that increasing the ambient temperature results in the growth of both the flame speed and the region of existence of the travelling wave solutions in the parameter space. The linear stability of the travelling wave solution is investigated analytically by using the matched asymptotic expansion method and numerically by employing the Evans function approach. We demonstrate that by increasing the ambient temperature the stability of the propagating wave can also be increased.

KEY WORDS: combustion waves, heat loss, ambient temperature, Evans function

AMS subject classification: 35K57, 80A25

1. Introduction

The reaction-diffusion models describing the propagation of combustion waves have been the subject of ongoing study for sometime [1]. Much effort

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has been invested into understanding the properties of the stationary propagating fronts and their stability. The main focus of this paper is the investigation of the effect of nonzero ambient temperature on the properties and stability of the steady planar combustion waves with respect to pulsating (longitudinal) perturbations. The latter allows a one-dimensional formulation of the propagation problem and is convenient for both analytical and numerical treatment. It is also a physically reasonable problem to investigate, since pulsating instabilities represent a distinct mechanism for the loss of stability of the travelling wave solution [1]. Indeed, as shown in [2] and [3], the geometrical parameters of the system can be chosen in such way that the transverse instabilities can be excluded. Moreover, pulsating and cellular instabilities manifest themselves in different disjoint domains in the space of the reaction parameters [1]. Namely, pulsating instabilities have been observed for the values of the Lewis number (the ratio of the diffusion rates of mass and heat) greater than one, whereas cellular instabilities are known to appear for the values of the Lewis number smaller than one [1]. For this reason, the mechanism leading to cellular waves is also excluded by an appropriate choice of the parameters (i.e. for Lewis number greater than one which is the case in this paper). Finally, the experimental observation of the pulsating waves in Self-propagating High-temperature Synthesis (SHS) [1,4–6] is probably the best evidence that, under certain conditions, longitudinal instabilities are the dominant way in which a planar wave can lose its stability. As noted in [6] pulsating combustion frequently occurs in experiments on SHS leading to layered structure of the resulting materials which is often an undesirable effect. This makes the study and control of the transition from steady to pulsating combustion a subject of great practical importance.

The one-dimensional formulation of the propagation problem which is used throughout of this paper is not new. It has been used in a number of papers to predict and investigate phenomena such as pulsating waves [3,7,8], period doubling in the oscillations of speed [9,10] and chaotic flames [3,11–13]. The standard model describing the propagation of combustion waves involves the reaction-diffusion equations for two components: the temperature, and the amount of fuel with Arrhenius reaction terms giving the strong nonlinear dependence of the reaction rate on the temperature [2,14].

The analytical investigation of the problem is usually based upon the application of the matched asymptotic expansion method, which is valid in the limit of large activation energy. This approach allows one to describe the properties of the steady solution, such as speed in a consistent way for both adiabatic [2,15] and nonadiabatic [16,17] flames. However, the analysis of the linear stability problem using matched asymptotic expansion method encounters difficulties known as the closure problem [2,14]. In the adiabatic case there are several different ways to circumvent this, including the truncated series or using the nearly equidiffusional approximation [2]. The latter method requires the Lewis number to be asymptotically close to unity whereas the truncated models were derived for general values of the Lewis number. In the nonadiabatic case the closure problem can be circumvented only in the nearly equidiffusional limit [16]. Asymptotic methods are able to give correct qualitative results. However, the ambient temperature is not present explicitly in the final asymptotic expressions of the aforementioned papers and it is not clear how it effects the properties and stability of the travelling wave.

One of the most straight forward control parameters in experiments is the ambient temperature. The effect of the ambient temperature on the combustion processes has been usually investigated while considering ignition problems [18,19]. However this issue has not been investigated systematically for the propagation problem although there is evidence from experimental data showing the influence of ambient temperature on the properties and the stability of the steady planar combustion waves in SHS [4–6].

As a consequence of the above discussion, it is clear that an investigation into the effects of varying the ambient temperature, for the case of propagating waves, should be undertaken. Therefore in this paper we undertake such a study and show how nonzero ambient temperature affects the process of propagation of the steady combustion wave both analytically and numerically. In our analytical studies, using the matched asymptotic expansion method, we choose the nondimensional variables in such way as to obtain the explicit dependencies of the properties of the travelling waves on the ambient temperature. Our numerically investigation utilized the shooting and relaxation methods developed in our previous papers [20,21].

The rest of this paper is essentially divided into two parts. In the first part we investigate how the properties of the steady propagating combustion waves are effected by the nonzero ambient temperature. The stability analysis of the travelling combustion waves is the subject of the second part of the paper. Finally, in the concluding section, we summarize the results and discuss the application of our findings to SHS experiments.

2. Model

We consider a premixed fuel in one dimension. It is assumed that the rate of exothermic combustion is well described by the Arrhenius law. In non-dimensional coordinates, the equations governing this process can be found in [10] and are given as

$$u_t = u_{xx} + v\mathrm{e}^{-1/u} - \ell(u - u_\mathrm{a}), \qquad v_t = \tau v_{xx} - \beta v\mathrm{e}^{-1/u}, \tag{1}$$

where u and v are non-dimensional temperature and the mass fraction of fuel, respectively; τ is the inverse Lewis number (the ratio of the diffusion rates of mass and heat); β is the ratio of the activation energy to heat release; ℓ is the heat loss coefficient from fuel to surroundings; u_a is the ambient temperature.

We consider the ambient temperature to be a small quantity in the current non-dimensional variables. This approximation is feasible for such propagation combustion wave problems since in the SHS experiments described in [6] the dimensionless ambient temperature defined above usually varies between 0 and 0.03. Parameter τ varies from zero (Lewis number $\to \infty$), for solid fuel, to unity (Lewis number equals to 1), for gaseous fuels. The parameter β is of the order of unity or larger. The heat loss ℓ is a parameter, which can be manipulated in the laboratory. In order for stationary solutions to exist, ℓ must be taken sufficiently small as will be seen in the next section. We consider system (1) subject to the following boundary conditions

$$\begin{aligned} &u(x,t) \to u_{\mathrm{a}},\ v_x(x,t) \to 0 \ \text{ as } \ x \to -\infty, \\ &u(x,t) \to u_{\mathrm{a}},\ v_x(x,t) \to 1 \ \text{ as } \ x \to +\infty. \end{aligned} \tag{2}$$

In other words on the right hand boundary we have cold ($u = u_{\mathrm{a}}$) and unburned ($v = 1$) state, whereas the opposite limit corresponds to partly burned ($v = \sigma$) state, where the temperature is cooling down to the ambient value ($u = u_{\mathrm{a}}$). Here σ is a constant which represents the residual amount of fuel left after the combustion wave ($\sigma = 0$ for the adiabatic case).

We will seek the solution of (1) in a form of the front travelling with a constant speed c, that is, we introduce a moving coordinate frame $\xi = x - ct$ and so

$$u(x,t) = u(\xi), \qquad v(x,t) = v(\xi). \tag{3}$$

After substituting (3) into (1) it is easy to obtain two second order ordinary differential equations

$$u_{\xi\xi} + cu_{\xi} + v\mathrm{e}^{-l/u} - \ell(u - u_{\mathrm{a}}) = 0, \qquad \tau v_{\xi\xi} + cv_{\xi} - \beta v\mathrm{e}^{-l/u} = 0, \tag{4}$$

with boundary conditions

$$\begin{aligned} &u = u_{\mathrm{a}},\ v_{\xi} = 0, \ \text{ as } \xi \to -\infty, \\ &u = u_{\mathrm{a}},\ v = 1, \quad \text{as } \xi \to +\infty. \end{aligned} \tag{5}$$

3. Matched asymptotic expansion method for a travelling wave solution

For the matched asymptotic analysis it is convenient to rewrite equations (4) describing the travelling wave solution in a symmetric form by introducing new variables $\tilde{u} = \beta u$, $\tilde{v} = v$ and a coordinate in the moving frame scaled by the speed of the front $\tilde{\xi} = c(x - ct)$. This yields the equations

$$\begin{aligned} &u_{\xi\xi} + u_{\xi} + \beta^2 Qv \exp(\beta(1 - 1/u)) - h(u - u_{\mathrm{a}})/\beta = 0, \\ &\tau_{\xi\xi} + v_{\xi} + \beta^2 Qv \exp(\beta(1 - 1/u)) = 0, \end{aligned} \tag{6}$$

where $Q = \beta^{-1}c^{-2}e^{-\beta}$ is a flame speed eigenvalue and $h = \beta\ell c^{-2}$ is the scaled heat-loss parameter. For ease and clarity tildes are omitted in equations (6) and in the rest of this section, unless stated otherwise.

The system (6) has to be solved subject to the boundary conditions

$$\begin{aligned} &u(\xi) = u_{\mathrm{a}},\ v_\xi(\xi) = 0, \quad \text{as} \quad \xi \to -\infty, \\ &u(\xi) = u_{\mathrm{a}},\ v(\xi) = 1, \quad \text{as} \quad \xi \to +\infty. \end{aligned} \tag{7}$$

Here we assume that the ambient temperature is much smaller than the burning temperature and we consider that $u_{\mathrm{a}} \sim O(\beta^{-1})$.

It is known [16] that the structure of the solution to equations (6) consists of three regions: the preheat region, where the fuel has not been consumed; the inner region, where the reaction occurs; and the product region, where almost all fuel has been burned and the temperature decays gradually to the ambient value.

According to the matched asymptotic expansion method the solution is first sought in the form of an infinite series (with β^{-1}being a small parameter) in each of these three zones, after which the matching conditions on the boundaries of these regions are imposed. Here we skip the details of derivation as they are similar to the ones presented in [16] and proceed to state only the main results. The resulting formula for the nonadiabatic speed of the front is given as

$$c^2 = c_{\mathrm{ad}}^2 \mathrm{e}^{-2\beta\ell c^{-2+u_{\mathrm{a}1}}}, \tag{8}$$

where $c_{\mathrm{ad}} = \sqrt{2/\tau_0\beta}\mathrm{e}^{-\beta/2}$ is the adiabatic flame speed and $u_{\mathrm{a}} = \beta^{-1}u_{\mathrm{a}1}$. In the original unscaled variables equation (8) can be expressed as

$$c^2 = c_{\mathrm{ad}}^2 \mathrm{e}^{-2\beta\ell c^{-2+u_{\mathrm{a}}\beta^2}}. \tag{9}$$

In contrast to the result obtained in [16] for the nonadiabatic flame speed this formula contains u_{a} in a explicit way.

The asymptotic analysis described above is valid for arbitrary values of τ (the inverse Lewis number) and $\beta \gg 1$. It has also been assumed that there is no fuel left behind the front in the product zone. The expression for the speed of the front (9) reveals a significant difference in comparison to the adiabatic problem. Namely, for fixed parameter τ there is a critical value $\beta_e\ell_e$ such that the steady propagating front solutions exist for $\beta\ell$ less than the critical value, and do not exist for larger values of $\beta\ell$. This effect is called extinction. Sometimes this event is referred to as the saddle-node bifurcation or turning point [22]. We use subscript 'e' to denote the critical parameter values, which can be found from (9) by solving the equation

$$\frac{\partial(\beta\ell)}{\partial c} = 2c\ln(c_{\mathrm{ad}}/c) - c + u_{\mathrm{a}1}c = 0. \tag{10}$$

This results in the equation for the critical heat loss

$$\ell_e(\beta, \tau) = \frac{e^{u_a\beta^2 - 1 - \beta}}{\tau\beta^2} \tag{11}$$

in the original variables. Similar results were also obtained in [23]. For fixed values of β, τ and $\ell < \ell_e$ there exist two solutions with different speeds, a "fast" and a "slow" solution.

4. Numerical results

We solve the boundary value problem (4–5) numerically by using the shooting method to obtain the guess solution and then the results are corrected with a more accurate method; namely relaxation. It follows from (11), that the solution exists only if the heat loss is sufficiently small. On the other hand, in the product zone the rate of the exponential decay of the temperature to u_a is governed by ℓ. This implies that in order to determine the stationary front numerically one has to follow the slowly decaying tails in the product zone, followed by a very rapid jump of the temperature and the amount of fuel in the reaction zone, and finally relatively fast decay in the preheat zone. Therefore, in contrast to the adiabatic problem [18], we use a nonuniform mesh along the ξ coordinate. The fifth-order Runge-Kutta method with the adaptive step size control is employed for the shooting method. The method not only allows the estimation of the local error, which has been set to be 10^{-5} in our calculations, but at the same time it produces nonuniform grid on the interval of integration. The relaxation method has been modified in order to deal with the nonuniform grids. The stability analysis of the steady propagating combustion front carried out in the following sections is based on the accuracy of our approximation of the solution to equation (4). The relaxation routine allows us to control the average local correction made on each iteration step. The solution is considered to be resolved if the correction is less than 10^{-15}.

Here we note that there is a difficulty in treating the boundary conditions (5) known as the 'cold-boundary' problem [24]. Indeed when $|\xi|$ tends to infinity the reaction terms in (4) do not vanish since they are proportional to $e^{-1/u}$ which therefore tends to e^{-l/u_a}. One approach to overcome this difficulty is to introduce an artificial "cut-off" condition [24]: the rate of chemical reaction is taken to be equal to zero at the ambient temperature. The "cut-off" condition is important as it enables rigorous mathematical statement of the propagation problem [24]. However for the values of ambient temperature $0 < u_a < 0.02$ considered in this paper e^{-l/u_a} varies from 0 to approximately 10^{-22}. Consequently this does not impact on the numerical scheme. Therefore the "cold-boundary" problem mentioned above does not require any special treatment here.

Firstly, we investigate the dependence of the speed of the front on the ambient temperature. In figure 1 the flame speed is shown as a function of the logarithm of the heat loss for fixed values of β and τ and different values of u_a

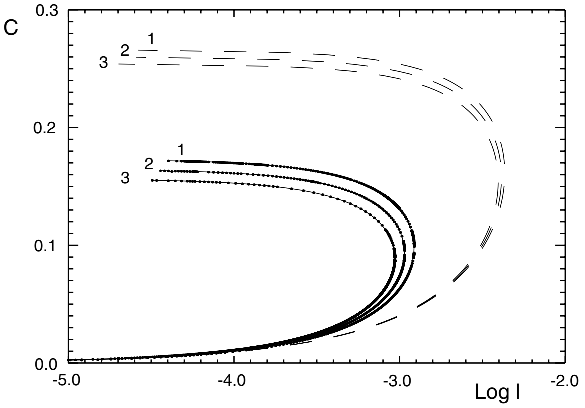

Figure 1. Speed of the travelling front as a function of ℓ for $\tau = 0.5$, $\beta = 3$, and different values of ambient temperature u_a: curves 1 correspond to $u_a = 0.01$, curves 2 to $u_a = 0.005$, curves 3 to $u_a = 0.0$. Dots connected with solid lines represent the numerical results obtained with shooting and relaxation methods, whereas dashed lines were plotted according to the asymptotic result (9).

as indicated in the caption. We also plotted the prediction obtained with the asymptotic method. Quantitatively the difference between numerical and asymptotic approaches is significant which justifies doing the numerical calculations away from the region where the asymptotic method is reasonable. Qualitatively they both predict the same behaviour: as we increase the ambient temperature for fixed values of β, τ , and ℓ the speed of the front rises and this is clearly seen in figure 2, where we plot the dependence of the speed of combustion wave on

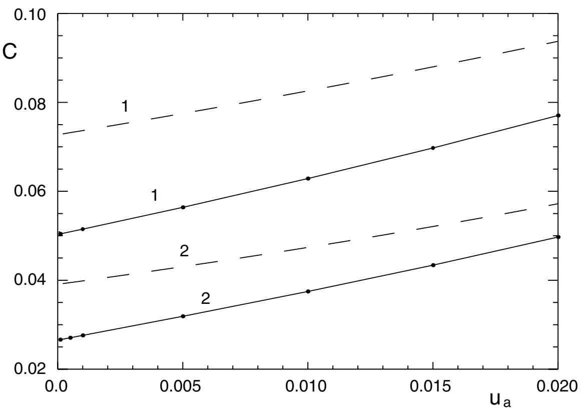

Figure 2. Dependence of the speed c of combustion wave on ambient temperature u_a for $\tau = 0.5, \ell = 10^{-5}$, and different values of β: curves 1 for $\beta = 6$ and curves 2 for $\beta = 5$. Dots connected with solid lines represent numerical results and dashed lines were plotted according to the asymptotic result (9).

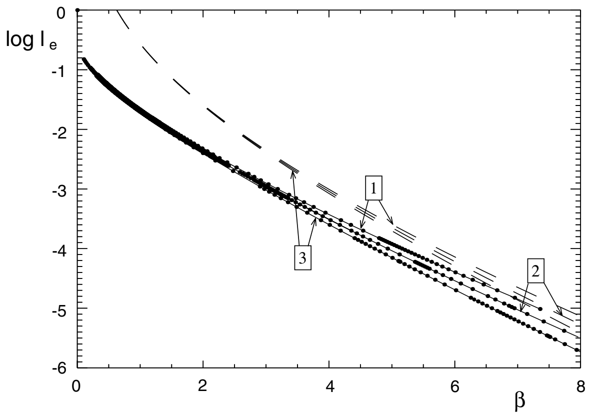

Figure 3. Dependence of the extinction value of ℓ on β for $\tau = 0.5$ and different values of u_a: curves 1 for $u_a = 0.01$, curves 2 for $u_a = 0.005$, and curves 3 for $u_a = 0$. Dots connected with solid lines correspond to numerical results. Dashed lines were plotted according to the asymptotic result (11).

the ambient temperature for fixed τ and ℓ and different values of β. This result is also in qualitative agreement with the experimental data [6].

It is interesting to note that as we increase u_a the point of extinction moves towards larger values of heat loss therefore increasing the region of parameter values for which the travelling wave solution exists. This effect is clearly seen in figure 3 where we plot the extinction values of heat loss as a function of β for $\tau = 0.5$ for different values of u_a. It is also in qualitative agreement with the asymptotic results (11), which are shown with the dashed lines. As is seen from these results the predictions from the numerical and the asymptotic approaches improve for larger values of β.

5. Stability of travelling front

As a first step in the analysis of the travelling wave stability we linearize (1) around the front solution (3)

$$u(x,t) = u(\xi) + \varphi(\xi,t), \qquad v(x,t) = v(\xi) + \chi(\xi,t), \tag{12}$$

where φ and χ are linear perturbation terms. After substitution of (12) into (1) it is straightforward to derive

$$\begin{pmatrix} \partial\varphi/\partial t \\ \partial\chi/\partial t \end{pmatrix} = \hat{L}\begin{pmatrix} \varphi \\ \chi \end{pmatrix}, \tag{13}$$

where

$$\hat{L} = \begin{pmatrix} \partial_\xi^2 + vu^{-2}\mathrm{e}^{-l/u} + c\partial_\xi - \ell & \mathrm{e}^{-l/u} \\ -\beta v u^{-2}\mathrm{e}^{-1/u} & \tau\partial_\xi^2 - \beta\mathrm{e}^{-1/u} + c\partial_\xi \end{pmatrix}. \tag{14}$$

The stability of the travelling front is then defined from the spectrum of $\hat{L}$. It is easy to show that the essential spectrum of this operator always lies in the left half plane (see [25,26]). Therefore, the discrete spectrum is solely responsible for the transition to instability. We will seek the solution of (13) of the form

$$\varphi(\xi, t) = \varphi(\xi)\mathrm{e}^{\lambda t}, \quad \chi(\xi, t) = \chi(\xi)\mathrm{e}^{\lambda t}, \tag{15}$$

where λ is a spectral parameter (in combustion literature it is sometimes referred to as the growth rate eigenvalue). Substituting (15) into (13) and introducing a vector with components $z_1 = \varphi, z_2 = \varphi_\xi, z_3 = \chi, z_4 = \chi_\xi$ we obtain a system of ODEs in the form

$$\dot{\mathrm{z}} = \mathrm{Az}, \tag{16}$$

where

$$A(\xi, \lambda) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \lambda + \ell - vu^{-2}\mathrm{e}^{-1/u} & -c & -\mathrm{e}^{-1/u} & 0 \\ 0 & 0 & 0 & 1 \\ \beta\tau^{-1}vu^{-2}\mathrm{e}^{-1/u} & 0 & \tau^{-l}(\lambda + \beta\mathrm{e}^{-l/u}) & -\tau^{-1}c \end{pmatrix}. \tag{17}$$

We use equation (16) to investigate the stability of the travelling front. Following [27], we will say that the travelling front is linearly unstable if, for some fixed complex λ with $\mathrm{Re}(\lambda) > 0$, there exists a solution of (16) which decays exponentially as $\xi \to \pm\infty$. We will refer to this λ as an eigenvalue and to the corresponding solution as an eigenmode.

6. Matched asymptotic analysis for the linear stability problem

The linear stability problem (13–14) is considered in [16] and [17] in the framework of the matched asymptotic expansion method. However the ambient temperature does not appear in the final results of these papers. Here we carry out asymptotic analysis in such a way that u_a is included explicitly via the dependence of stationary solution on u_a. It is interesting to note that the ambient temperature cancels out of the main result for the stability analysis. In particular the growth rate eigenvalue is independent of u_a, but is a function of the other parameters of the problem (here we imply scaled parameters h, Q, and τ). This is due to the nearly equidiffusional approximation (i.e. $\tau = 1+\sum_{i=1}^{\infty} \tau_i\beta^{-i}$), which has to be used in order to circumvent the closure problem [14]. As a result the growth rate eigenvalue is given by an expression equivalent to the one derived in [16,17]. In order to avoid possible repetition we skip the derivation and and refer the reader to these papers for the details.

The main result of the stability analysis using the matched asymptotic expansion method is given by the formula

$$\Gamma^2 - h(\Gamma + 1) = \frac{\tau_1}{4}(1 - \Gamma), \tag{18}$$

where $\Gamma = \sqrt{1 + 4\lambda}$.

A similar equation relating λ to h and τ_1 was derived earlier in [16] and [17] for the equivalent models. It is worthwhile to note that the point of extinction corresponds to $h = 1/2$. When $h < 1/2$ this corresponds to the fast branch whereas when $h > 1/2$ refers to the slow branch. According to [16] there is always one solution of (18) with $\mathrm{Im}(\lambda_0) = 0$ and $\mathrm{Re}(\lambda_0) > 0$ for the slow branch ($h > 1/2$), therefore the slow waves are always unstable (or in other words, there is always a single point of the discrete spectrum located in the right half plane). The stability of the fast solution branch depends on the parameter τ_1.

When $-6 < \tau_1 < 0$ the fast solutions ($h < 1/2$) are stable. The transition to instability occurs at the point of extinction: as we cross the extinction value $h = 1/2$ moving from the fast to the slow solution branch, one of the roots of (18) moves from the left half plane to the right half plane along the real axis giving rise to monotonic instability.

If $\tau_1 < -6$ the fast solution branch losses stability via a Hopf bifurcation when we cross the curve $h_2(\tau_1) = \tau_1/4 - 1 + \sqrt{3 - \tau_1}$ in the parameter plane (h, τ_1). In other words the fast waves are stable if $h < h_2(\tau_1)$, however when we cross the critical value $h_2(\tau_1)$ two complex conjugate roots of (18) cross the imaginary axis giving rise to an oscillatory instability.

Expression (18) does not contain u_a explicitly. However, if we return to the original parameters β, τ, ℓ and u_a in (18) such a dependence will occur due to the fact that the speed (9) of the travelling wave is a function of u_a.

7. Numerical results for the linear stability problem

In order to investigate the linear stability of the travelling wave solutions, eigenvalues and eigenmodes of the problem (16–17) have to be found. We solve the linear stability problem (16–17) numerically by using the Evans function method, which is described in detail in [18]. It uses the compound matrix method to eliminate the stiffness of the linear stability problem (16–17). However, there is a difference between the algorithm employed in this paper and the algorithm described in [18], for the adiabatic case and $u_a = 0$. This is due to the nonuniform grid which we use to approximate the stationary solution of the nonadiabatic problem (4). The spectral problem (16) is integrated numerically by means of the fifth-order Runge-Kutta method with the adaptive stepsize control as in section 4. The numerical scheme for solving (16) requires the stationary solution, which appears in the equations explicitly, to be approximated at arbitrary values of coordinate ξ In other words, grids for the systems (4) and (16) do not coincide. The Neville's algorithm [28] for constructing the interpolation polynomial is utilized to find $u(\xi)$ and $v(\xi)$ for any value of ξ inside of the integration interval. The Neville's scheme uses the values of $u(\xi)$ and $v(\xi)$ (obtained by numerically solving (4)) at four grid points closest to ξ and allows the control of the error made while interpolation. In our calculations this error has been of the order of 10^{-13}. The error of the numerical integrator has been set to be 10^{-5} as in section 4.

The Evans function method allows us to locate the eigenvalues of the problem (16–17) on the complex plane. We are able to detect whether the travelling solution is stable or not, and to investigate the scenarios of transition to instability. This includes determining the location of the neutral stability boundary in the parameter space. The results of this analysis are summarized in figure 4. Here we plot the critical parameter values for the Hopf bifurcation (thin lines) and extinction (thick lines) on the plane of parameters ℓ vs. β for $\tau = 0.5$ and different values of ambient temperature as indicated in the figure caption. For each value of u_a the region of parameters for which the travelling solution exists is located below the curve representing the extinction. For any parameter value from this region there are two travelling solution: fast and slow. For slow solution branch there is always a point of the discrete spectrum located in the right half plane on the real axis (this can be shown numerically by using the Nyquist plot technique [18]). Therefore in the discussion that follows we consider only the

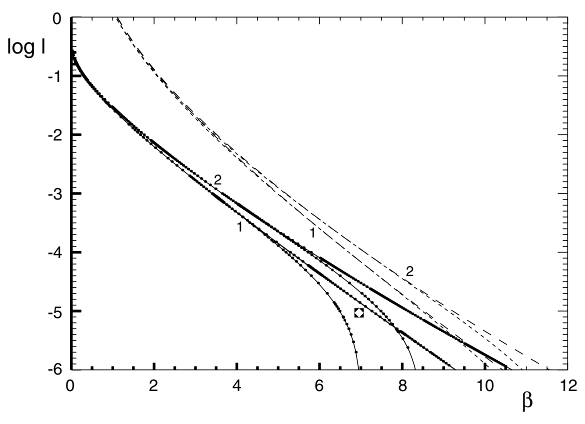

Figure 4. Stability diagram on the parameter plane ℓ vs. β for $\tau = 0.5$ and different values of ambient temperature u_a: curves 1 for $u_a = 0$ and curves 2 for $u_a = 0.01$. Dots connected with solid lines represent numerical results. Dashed lines were plotted according to (11) and (18). In both cases thin lines represent critical parameter values for Hopf bifurcation, whereas thick lines show critical parameter values for the extinction. The square corresponds to a particular choice of parameters, namely, $\ell = 10^{-5}$ and $\beta = 7$.

fast solution branch. In figure 4 solid lines represent the numerical results, whereas dashed lines are plotted based on the prediction of the asymptotic analysis. Qualitatively the results of both approaches agree. The critical curve for the Hopf bifurcation (showed with thin lines) lies below the curve representing extinction (showed with thick lines) and at some parameter values these curves intersect. For each value of u_a the travelling wave solution is stable for the parameters ℓ and β located below both the curve representing extinction and the curve representing Hopf bifurcation. For ℓ and β located between these curves the travelling wave solution exhibits oscillatory instability.

However the most interesting effect which can be seen in figure 4 is stabilization of the travelling wave with increasing the ambient temperature. Indeed both asymptotic and numerical results predict that the region of stability is larger for increasing values of u_a. For example if we take $\ell = 10^{-5}$ and $\beta = 7$ (this choice of parameters is shown with a square in figure 4, then the fast travelling wave solution is unstable for $u_a = 0$ (the point lies in the region of oscillatory instability in figure 4). However if we increase the initial temperature to $u_a = 0.01$ the solution becomes stable. In practice this implies that oscillatory regimes of wave propagation during the Self-Propagating High-Temperature Synthesis may be avoided by initial preheating of the sample.

8. Conclusions

In this paper we have undertaken an analytical and numerical investigation of the effect of nonzero ambient temperature on the linear stability and properties of the planar travelling nonadiabatic combustion wave. To the best of our knowledge, such an investigation has never been carried out previously.

The properties of the travelling wave are investigated analytically employing the matched asymptotic expansion approach and numerically using the shooting and relaxation methods. Quantitatively the difference between numerical and asymptotic approaches is significant. However, qualitatively they both predict the same behaviour: as we increase the ambient temperature for fixed values of β, τ, and ℓ the speed of the front rises. This result is also in qualitative agreement with the experimental data for the SHS experiments [6]. The other interesting effect which is confirmed qualitatively by both approaches is that increasing the ambient temperature results in the shift of the point of extinction towards larger values of heat loss. This increases the region of parameter values for which the travelling wave solution exists.

The linear stability problem for the travelling wave solution is solved analytically by using the matched asymptotic expansion method and numerically by employing the Evans function approach. Once again both methods predict qualitatively equivalent result: increasing the ambient temperature shifts the region of oscillatory instability towards larger values of β increasing the region of stability of the travelling wave. There is an indication [4] that such an effect of stabilization was observed in SHS experiments.

The stabilization of the combustion wave by increasing the ambient temperature has practical implications, and can be employed to avoid the oscillatory regime of wave propagation which is often the dominant way for the loss of stability in SHS experiments [6]. Consequently the undesirable layered structure of the resulting materials from pulsating waves can be avoided altogether.

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